Midterm Test 3: November 17, 2004, Version A, Solutions.

1. (2 points) Using the Midpoint Rule with $n = 4$, approximate $\displaystyle\int_0^1 \cos(x^2)\,dx$.

A. 0.9089 **B.** 3.6356 **C.** 1.2295
D. 0.3074 **E.** 0.8437 **F.** 0.4609

$\frac{1}{4}[\cos((\frac{1}{8})^2) + \cos((\frac{3}{8})^2) + \cos((\frac{5}{8})^2) + \cos((\frac{7}{8})^2)] = 0.9089.$ Answer is **A**

2. (2 points) A table of values for a decreasing function $f(x)$ is given below. Use the data in the table to find an overestimate and an underestimate for $\displaystyle\int_0^6 f(x)\,dx$.

x	0	1	2	3	4	5	6
$f(x)$	23	17	16	12	11	9	7

A. overestimate is 88, underestimate is 65 **B.** overestimate is 88, underestimate is 72
C. overestimate is 95, underestimate is 72 **D.** overestimate is 95, underestimate is 65
E. overestimate is 95, underestimate is 79 **F.** overestimate is 88, underestimate is 79

Function is decreasing, so left-hand endpoints give overestimate; right-hand endpoints give underestimate. Overestimate is $23 + 17 + 16 + 12 + 11 + 9 = 88$ and underestimate is $17 + 16 + 12 + 11 + 9 + 7 = 72$.
Answer is **B**

3. (2 points) Evaluate the definite integral $\displaystyle\int_1^2 x(x^3 - 3)\,dx$.

A. 12 **B.** 0.875 **C.** 3.2
D. 13 **E.** 1.7 **F.** -2.625

$$\int_1^2 x(x^3 - 3)\,dx = \int_1^2 (x^4 - 3x)\,dx$$
$= (\frac{1}{5}x^5 - \frac{3}{2}x^2)|_1^2 = (1/5)(32) - (3/2)(4) - ((1/5) - (3/2)) = (1/5)(31) - (3/2)(3) =$

1.7.
Answer is **E**

4. (2 points) Consider the following statements about integrals. Circle the letters of statements that are true and cross out the letters of statements that are false. Assume the functions are continuous and that the integrals exist.

A. $\displaystyle\int_a^b f'(x)\,dx \;=\; f(b)-f(a)$

B. $\displaystyle\int_a^b f(x)\,g(x)\,dx \;=\; \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

C. $\displaystyle\int_a^b 3\,f(x)\,dx \;=\; 3\int_a^b f(x)\,dx$

D. $\displaystyle\int_a^b x^2\,f(x)\,dx \;=\; x^2\int_a^b f(x)\,dx$

Circle **A** and **C**; cross out **B** and **D**.
Marking Scheme: for one mistake, give 1 out of 2; for two or more mistakes, give 0.

5. (2 points) a) If $\displaystyle g(x)=\int_0^x \sqrt{1+t^3}\,dt$, what is $g'(x)$?

b) What is $\displaystyle\frac{d}{dx}\int_0^{x^2}\sqrt{1+t^3}\,dt$?

a) $g'(x)=\sqrt{1+x^3}$

Marking Scheme: give 0.5 out of 1 if the function is correct, but the variable is wrong.

b) $\displaystyle\frac{d}{dx}\int_0^{x^2}\sqrt{1+t^3}\,dt=\sqrt{1+(x^2)^3}\,\frac{d}{dx}(x^2)=2x\sqrt{1+x^6}$
Marking Scheme: give 0.5 out of 1 if the variable is wrong; give 0.5 if they have $2x\sqrt{1+x^3}$.

6. (2 points) a) Find the most general antiderivative of the function $f(x)=7x^{-1}+3\sec^2(x)+4$.
a) $F(x)=7\ln|x|+3\tan x+4x+C$.
Marking Scheme: 0.25 for each correct term.

b) Find the antiderivative (indefinite integral) $\displaystyle\int\frac{3x^2-4}{\sqrt{x}}\,dx$.

Marking Scheme: 0.25 for each correct term in the simplification; 0.25 for each correct antiderivative. Do not deduct if they forget the C. If the simplification is incorrect, but then they antidifferentiate properly (for their mistake), give 0.5 out of 1. Give 0 if their method is mathematically incorrect.

7. (2 points) Find a function $f(t)$ such that $f(0) = 5$ and $f'(t) = 3\sin(t) - 4e^t$.
$f(t) = -3\cos(t) - 4e^t + C$. $f(0) = -3 - 4 + C = 5$, so $C = 12$ and $f(t) = -3\cos(t) - 4e^t + 12$.
Marking Scheme: 0.5 for $-3\cos(t)$; 0.5 for $-4e^t$; 0.5 for $C = 12$ and 0.5 for final $f(t)$. If they forget the C, give 1 out of 2 if the antidifferentiation is correct.

8. (2 points) Evaluate the definite integral $\displaystyle\int_{0.2}^{0.8} \left(\frac{4}{\sqrt{1-x^2}} + 3 \right) dx$.

$$\int_{0.2}^{0.8} \left(\frac{4}{\sqrt{1-x^2}} + 3 \right) dx$$
$= (4\arcsin x + 3x)|_{0.2}^{0.8} = 4\arcsin(0.8) + 3(0.8) - (4\arcsin(0.2) + 3(0.2)) = 4.7037$
Marking Scheme: give 1 out of 2 for the correct antidifferentiation; 1 for the correct evaluation.

Version 2, solutions

1. (2 points) Using the Midpoint Rule with $n = 4$, approximate $\displaystyle\int_0^1 \sin(x^2)\, dx$.

A. 0.9089	**B.** 3.6356	**C.** 1.2295
D. 0.3074	**E.** 0.8437	**F.** 0.4609

$\frac{1}{4}[\sin((\frac{1}{8})^2) + \sin((\frac{3}{8})^2) + \sin((\frac{5}{8})^2) + \sin((\frac{7}{8})^2)] = 0.3074.$
Answer is **D**

2. (2 points) A table of values for a decreasing function $f(x)$ is given below. Use the data in the table to find an overestimate and an underestimate for $\displaystyle\int_0^6 f(x)\, dx$.

x	0	1	2	3	4	5	6
$f(x)$	31	27	23	19	13	10	6

A. overestimate is 123, underestimate is 92
B. overestimate is 129, underestimate is 92
C. overestimate is 123, underestimate is 104
D. overestimate is 129, underestimate is 98
E. overestimate is 129, underestimate is 104
F. overestimate is 123, underestimate is 98

Function is decreasing, so left-hand endpoints give overestimate; right-hand endpoints give underestimate. Overestimate is $31+27+23+19+13+10 = 123$ and underestimate is $27+23+19+13+10+6 = 98$.
Answer is **F**

3. (2 points) Evaluate the definite integral $\displaystyle\int_1^2 x(x^3-2)\,dx$.

A. 12 **B.** 0.875 **C.** 3.2
D. 13 **E.** 1.7 **F.** -2.625

$$\int_1^2 x(x^3-2)\,dx = \int_1^2 (x^4-2x)\,dx$$
$= (\frac{1}{5}x^5 - x^2)|_1^2 = (1/5)(32) - (4) - ((1/5) - 1) = (1/5)(31) - 3 = 3.2.$
Answer is **C**

4. (2 points) Consider the following statements about integrals. Circle the letters of statements that are true and cross out the letters of statements that are false. Assume the functions are continuous and that the integrals exist.

A. $\displaystyle\int_a^b 5\,f(x)\,dx \;=\; 5\int_a^b f(x)\,dx$

B. $\displaystyle\int_a^b f(x)\,g(x)\,dx \;=\; \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

C. $\displaystyle\int_a^b x^3\,f(x)\,dx \;=\; x^3\int_a^b f(x)\,dx$

D. $\displaystyle\int_a^b f'(x)\,dx \;=\; f(b)-f(a)$

Circle **A** and **D**; cross out **B** and **C**.
Marking Scheme: for one mistake, give 1 out of 2; for two or more mistakes, give 0.

5. (2 points) a) If $g(x) = \displaystyle\int_0^x \frac{1}{\sqrt{1+t^3}}\,dt$, what is $g'(x)$?

b) What is $\displaystyle\frac{d}{dx}\int_0^{x^2} \frac{1}{\sqrt{1+t^3}}\,dt$?

a) $g'(x) = \dfrac{1}{\sqrt{1+x^3}}$

Marking Scheme: give 0.5 out of 1 if the function is correct, but the variable is wrong.

b) $\displaystyle\frac{d}{dx}\int_0^{x^2} \frac{1}{\sqrt{1+t^3}}\,dt = \frac{1}{\sqrt{1+(x^2)^3}}\,\frac{d}{dx}(x^2) = \frac{2x}{\sqrt{1+x^6}}$

Marking Scheme: give 0.5 out of 1 if the variable is wrong; give 0.5 if they have $2x/\sqrt{1+x^3}$.

6. (2 points) a) Find the most general antiderivative of the function $f(x) = 3x^{-1} + 4\csc^2(x) + 2$.

b) Find the antiderivative (indefinite integral) $\displaystyle\int \frac{4x^2-3}{\sqrt{x}}\,dx$.

a) $F(x) = 3\ln|x| - 4\cot x + 2x + C$.

Marking Scheme: 0.25 for each correct term. Do <u>not</u> deduct if they forget the absolute value bars in the natural log.

b) $\displaystyle\int \frac{4x^2-3}{\sqrt{x}}\,dx = \int (4x^{3/2} - 3x^{-1/2})\,dx = \tfrac{8}{5}x^{5/2} - 6x^{1/2} + C$.

Marking Scheme: 0.25 for each correct term in the simplification; 0.25 for each correct antiderivative. Do <u>not</u> deduct if they forget the C. If the simplification is incorrect, but then they antidifferentiate properly (for their mistake), give 0.5 out of 1. Give 0 if their method is mathematically incorrect.

7. (2 points) Find a function $f(t)$ such that $f(0) = 4$ and $f'(t) = 3\cos(t) + 6e^t$.

$f(t) = 3\sin(t) + 6e^t + C$. $f(0) = 6 + C = 4$, so $C = -2$ and $f(t) = 3\sin(t) + 6e^t - 2$.

Marking Scheme: 0.5 for $3\sin(t)$; 0.5 for $6e^t$; 0.5 for $C = -2$ and 0.5 for final $f(t)$. If they forget the C, give 1 out of 2 if the antidifferentiation is correct.

8. (2 points) Evaluate the definite integral $\displaystyle\int_1^5 \left(\frac{5}{1+x^2} + 7\right) dx$.

$$\int_1^5 \left(\frac{5}{1+x^2}+7\right)dx$$

$= (5\arctan x + 7x)|_1^5 = 5\arctan(5) + 7(5) - (5\arctan(1) + 7(1)) = 30.9400$

Marking Scheme: give 1 out of 2 for the correct antidifferentiation; 1 for the correct evaluation.